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# Democratic Neutrino Mixing and Radiative Corrections

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## Abstract

The renormalization effect on a specific ansatz of lepton mass matrices, arising naturally from the breaking of flavor democracy for charged leptons and that of mass degeneracy for light neutrinos, is studied from a superhigh energy scale $M_0 \sim 10^{13}$ GeV to the electroweak scale in the framework of the minimal supersymmetric standard model. We find that the democratic neutrino mixing pattern obtained from this ansatz may in general be instable against radiative corrections. With the help of similar flavor symmetries we prescribe a slightly different scheme of lepton mass matrices at the scale $M_0$, from which the democratic mixing pattern of lepton flavors can be achieved, after radiative corrections, at the experimentally accessible scales.

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Recently a number of models of lepton mass matrices have been proposed at low energy scales [1], at which their consequences on the spectrum of neutrino masses and the mixing of lepton flavors can directly be confronted with the robust Super-Kamiokande data on atmospheric and solar neutrino oscillations [2]. From the theoretical point of view, however, a phenomenologically-favored scheme of lepton mass matrices might only serve as the low-scale approximation of a more fundamental theory responsible for the lepton mass generation and flavor mixing at superhigh energy scales. It is therefore desirable to investigate the scale dependence of lepton mass matrices with the help of the renormalization-group equations. So far some attempts have been made in this direction [3–6].

In this Brief Report we aim to study whether the democratic neutrino mixing pattern, which is indeed a nearly bi-maximal mixing pattern of lepton flavors arising from the breaking of flavor democracy for charged leptons and that of mass degeneracy for light neutrinos, can be stable or not against the effect of radiative corrections from a superhigh energy scale $M_0 \sim 10^{13}$ GeV to the electroweak scale $M_Z$ in the framework of the minimal supersymmetric standard model. We find that the democratic neutrino mixing pattern at $M_0$ is no longer of the same form at $M_Z$. But it can still be obtained at low energy scales, if an additional term preserving the symmetry of flavor democracy is introduced to the original neutrino mass matrix at the scale $M_0$. Therefore the democratic neutrino mixing pattern at the experimentally accessible scales might hint at certain lepton flavor symmetries at a superhigh scale, at which viable models of lepton mass matrices can naturally be built.

Let us begin with a brief retrospection of the specific model of democratic neutrino mixing proposed first in Ref. [7] at low energy scales. The essential idea of this model is that the realistic textures of charged lepton and neutrino mass matrices might arise respectively from the breaking of $\mathrm{S(3)_L \times S(3)_R}$ and S(3) flavor symmetries:

$$
\begin{aligned}
M_l &= \frac{c_l}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \Delta M_l \; , \\
M_\nu &= c_\nu \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \Delta M_\nu \; ,
\end{aligned}
\tag{1}
$$

where $c_l$ and $c_\nu$ measure the corresponding mass scales of charged leptons and neutrinos. The explicit symmetry-breaking term $\Delta M_l$ is responsible for the generation of muon and electron masses, and $\Delta M_\nu$ is responsible for the breaking of neutrino mass degeneracy. The lepton flavor mixing matrix results from the mismatch between the diagonalization of $M_l$ and that of $M_\nu$, therefore its pattern depends crucially on the forms of $\Delta M_l$ and $\Delta M_\nu$. It has been shown in Ref. [7] that current data on solar and atmospheric neutrino oscillations seem to favor the following forms of $\Delta M_l$ and $\Delta M_\nu$:

$$
\begin{aligned}
\Delta M_l &= \frac{c_l}{3} \begin{pmatrix} -i\delta_l & 0 & 0 \\ 0 & i\delta_l & 0 \\ 0 & 0 & \varepsilon_l \end{pmatrix} \; , \\
\Delta M_\nu &= c_\nu \begin{pmatrix} -\delta_\nu & 0 & 0 \\ 0 & \delta_\nu & 0 \\ 0 & 0 & \varepsilon_\nu \end{pmatrix} \; ,
\end{aligned}
\tag{2}
$$

where $(\delta_l, \varepsilon_l)$ and $(\delta_\nu, \varepsilon_\nu)$ are dimensionless perturbation parameters of small magnitude. It is easy to obtain $m_\tau \approx c_l$, $m_\mu \approx 2|\varepsilon_l| m_\tau/9$, and $m_e \approx |\delta_l|^2 m_\tau^2/(27 m_\mu)$ in the lowest order approximation. As for neutrino masses, we have $m_1 = c_\nu(1-\delta_\nu)$, $m_2 = c_\nu(1+\delta_\nu)$, and $m_3 = c_\nu(1+\varepsilon_\nu)$. The simultaneous diagonalization of $M_l$ and $M_\nu$ leads to the lepton flavor mixing matrix $V$, which links the neutrino flavor eigenstates $(\nu_e, \nu_\mu, \nu_\tau)$ to the neutrino mass eigenstates $(\nu_1, \nu_2, \nu_3)$:

$$V = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{-2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix} + \Delta V \ , \tag{3}$$

where

$$\Delta V \ = \ i \ \xi_V \sqrt{\frac{m_e}{m_\mu}} \ + \ \zeta_V \frac{m_\mu}{m_\tau} \tag{4}$$

holds in the next-to-leading order approximation with

$$\xi_V = \begin{pmatrix} \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{-2}{\sqrt{6}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 \end{pmatrix} \ ,$$
$$\zeta_V = \begin{pmatrix} 0 & 0 & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} \\ \frac{-1}{\sqrt{12}} & \frac{-1}{\sqrt{12}} & \frac{1}{\sqrt{3}} \end{pmatrix} \ . \tag{5}$$

This is just a nearly bi-maximal lepton mixing pattern with large CP violation [7]. Neglecting the term $\Delta V$, which is remarkably suppressed by the small quantities $\sqrt{m_e/m_\mu} \approx 0.07$ and $m_\mu/m_\tau \approx 0.06$ [8], one often refers to $V$ as the democratic neutrino mixing pattern.

Now we prescribe the same ansatz of lepton mass matrices, as that introduced above, at superhigh energy scales. To be specific, we only consider the simple possibility that the typical mass scale of light Majorana neutrinos is determined via the conventional seesaw mechanism [9] by the mass of a heavy right-handed neutrino $M_0 \sim 10^{13}$ GeV; namely $c_\nu \sim v^2/M_0$ [10], where $v$ is the electroweak vacuum expectation value. The mass degeneracy of three active neutrinos is broken by the perturbative term $\Delta M_\nu$ in Eq. (1) at the scale $M_0$. Below this typical scale, the neutrino mass matrix $M_\nu$ and the charged lepton mass matrix $M_l$ have quite simple running behaviors in the framework of the standard electroweak model or its minimal supersymmetric extension. The relevant renormalization-group equations, which describe the radiative corrections to lepton mass matrices from the superhigh scale $M_0$ to the electroweak scale $M_Z$, have been derived by a number of authors in Refs. [3–6]. Subsequently we investigate whether the democratic neutrino mixing pattern $V$ in Eq. (3) is stable or not against radiative corrections in the framework of the minimal supersymmetric standard model (MSSM).

For simplicity we choose a specific flavor basis, in which the lepton mass matrix $M_l$ is diagonal (namely, $M_l$ is transformed into the diagonal form $\hat{M}_l = \text{Diag}\{m_e, m_\mu, m_\tau\}$) at the scale $M_0$. In this basis and at the same scale, the corresponding neutrino mass matrix takes the form $\hat{M}_\nu = V^* M_\nu V^\dagger$. Running $\hat{M}_l$ and $\hat{M}_\nu$ down to the scale $M_Z$ by use of the

renormalization-group equations in the framework of MSSM, one obtains the new lepton mass matrices $\hat{\mathbf{M}}_l$ and $\hat{\mathbf{M}}_\nu$. Obviously $\hat{\mathbf{M}}_l$ remains diagonal, but its mass eigenvalues are in general different from those of $\hat{M}_l$ due to radiative corrections [3,5]. At the scale $M_Z$, the form of $\hat{\mathbf{M}}_\nu$ reads explicitly as [4]

$$\hat{\mathbf{M}}_\nu \;=\; \left(I_g I_t^6\right) T_l \hat{M}_\nu T_l \tag{6}$$

with

$$T_l \;=\; \begin{pmatrix} I_e & 0 & 0 \\ 0 & I_\mu & 0 \\ 0 & 0 & I_\tau \end{pmatrix} , \tag{7}$$

in which $I_g$, $I_t$, and $I_\alpha$ (for $\alpha = e, \mu, \tau$) denote the corresponding evolution functions of the gauge couplings $g_1$ and $g_2$, the top-quark Yukawa coupling $f_t$, and the charged lepton Yukawa couplings $f_e$, $f_\mu$ and $f_\tau$:

$$\begin{aligned} I_g &= \exp\left[+\frac{1}{16\pi^2}\int_{\ln M_Z}^{\ln M_0}\left(\frac{6}{5}g_1^2(\chi)+6g_2^2(\chi)\right)\mathrm{d}\chi\right] \quad , \\ I_t &= \exp\left[-\frac{1}{16\pi^2}\int_{\ln M_Z}^{\ln M_0} f_t^2(\chi)\mathrm{d}\chi\right] \quad , \\ I_\alpha &= \exp\left[-\frac{1}{16\pi^2}\int_{\ln M_Z}^{\ln M_0} f_\alpha^2(\chi)\mathrm{d}\chi\right] \quad . \end{aligned} \tag{8}$$

Note that the power of $I_t$ in the expression of $\hat{\mathbf{M}}_\nu$ depends on the definition of $I_t$ in Eq. (8). The overall factor $(I_g I_t^6)$ in Eq. (6) does not affect the relative magnitudes of the matrix elements of $\hat{M}_\nu$. Only the matrix $T_l$, which amounts to the unity matrix at the energy scale $M_0$, can modify the texture of the neutrino mass matrix from $M_0$ to $M_Z$. The magnitude of $I_\tau$ may somehow deviate from unity, if $\tan\beta$ (the ratio of Higgs vacuum expectation values in the MSSM) takes large values. In contrast, $I_e \approx I_\mu \approx 1$ is an excellent approximation. Denoting $\kappa \equiv I_e/I_\tau - 1 \approx I_\mu/I_\tau - 1$, one arrives from Eq. (8) at

$$\kappa \approx \frac{m_\tau^2}{16\pi^2 v^2 \cos^2\beta} \ln\frac{M_0}{M_Z} \; . \tag{9}$$

It turns out that $\kappa \approx 0.03$ for $M_0 \sim 10^{13}$ GeV and $\tan\beta = 60$.

With the help of Eqs. (3) and (6), one can straightforwardly figure out the explicit expression of $\hat{\mathbf{M}}_\nu$ at the scale $M_Z$. To a good degree of accuracy, the small term $\Delta V$ of $V$ (namely, the small corrections from the ratios of charged lepton masses) is negligible in the calculation. We then make the transformations $V^\dagger \hat{\mathbf{M}}_l V \equiv \mathbf{M}_l$ and $V^{\mathrm{T}} \hat{\mathbf{M}}_\nu V \equiv \mathbf{M}_\nu$ at the scale $M_Z$. Of course $\mathbf{M}_l$ should have a quasi-democratic texture in the lowest order approximation, just as $M_l$. The neutrino mass matrix $\mathbf{M}_\nu$ takes the following form:

$$\mathbf{M}_\nu \approx \left(I_g I_t^6 I_\tau^2\right)\left[M_\nu + 2\kappa\left(\Omega_\nu - \Lambda_\nu\right)\right] \; , \tag{10}$$

where the $\kappa$-induced term signifies the renormalization effect, and the constant matrices $\Omega_\nu$ and $\Lambda_\nu$ read as

$$
\Omega_\nu \approx c_\nu \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} ,
$$

$$
\Lambda_\nu \approx \frac{c_\nu}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} . \tag{11}
$$

In obtaining Eqs. (10) and (11), we have neglected the small contributions of $\mathcal{O}(\kappa^2)$, $\mathcal{O}(\kappa\varepsilon_\nu)$ and $\mathcal{O}(\kappa\delta_\nu)$. Note that the factor $I_\tau^2$ in $\mathbf{M}_\nu$ comes from the product of two $T_l$ matrices on the right-hand side of $\hat{\mathbf{M}}_\nu$. Comparing $\mathbf{M}_\nu$ with $M_\nu$, we see that the diagonal texture of $M_\nu$ is not affected by the radiative correction term $\Omega_\nu$, which is also diagonal. The latter modifies three neutrino mass eigenvalues of $M_\nu$ with the same magnitude (proportional to $2\kappa c_\nu$). In contrast, the diagonal texture of $M_\nu$ is spoiled by the other radiative correction term $\Lambda_\nu$, which has a democratic form. As a consequence of the appearance of $\Lambda_\nu$ in $\mathbf{M}_\nu$, the corresponding lepton flavor mixing matrix $\mathbf{V}$, which arises from the mismatch between the diagonalization of $\mathbf{M}_l$ and that of $\mathbf{M}_\nu$ at the electroweak scale $M_Z$, may substantially deviate from the original flavor mixing matrix $V$ at the superhigh scale $M_0$. Unless $\kappa$ is negligibly small, the democratic neutrino mixing pattern is expected to be instable against radiative corrections.

The sensitivity of a nearly bi-maximal neutrino mixing pattern to the renormalization effect is of course not a big surprise [3,5]. However, it is not impossible to find out the appropriate textures of lepton mass matrices, which are essentially stable against radiative corrections [5,6]. As the democratic neutrino mixing pattern is only favored at the experimentally accessible energy scales, the question turns out to be whether there is a scheme of lepton mass matrices at the superhigh scale $M_0$, from which the democratic mixing pattern of lepton flavors can be obtained at the electroweak scale $M_Z$. We find that such a scheme of lepton mass matrices does exist and it is very similar to that discussed above.

To be specific, let us prescribe the new ansatz of lepton mass matrices at the scale $M_0$. We take the charged lepton mass matrix $M'_l$ to have the same form as $M_l$ in Eq. (1); namely, $M'_l = M_l$ has the $\mathrm{S(3)_L \times S(3)_R}$ flavor symmetry in the limit $\Delta M_l = 0$. The corresponding neutrino mass matrix $M'_\nu$ is a linear combination of $M_\nu$ in Eq. (1) and an additional term, which preserves the symmetry of flavor democracy:

$$
M'_\nu \;=\; M_\nu + c'_\nu \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} . \tag{12}
$$

Obviously $M'_\nu$ has the same S(3) flavor symmetry as $M_\nu$ in the limit $\Delta M_\nu = 0$. One can therefore see much similarity between the new ansatz and the old one. The coefficient $c'_\nu$ is a free parameter [6], but its value may be physically nontrivial, as we shall see below.

Following the same procedure as outlined above, we calculate the counterpart of $M'_\nu$ at the electroweak scale $M_Z$. We obtain

$$
\mathbf{M}'_\nu \approx \left( I_g I_t^6 I_\tau^2 \right) \left[ M'_\nu + 2\kappa \left( \Omega_\nu - \Lambda_\nu \right) \right] , \tag{13}
$$

where $\kappa$, $\Omega_\nu$, and $\Lambda_\nu$ have been given in Eqs. (9) and (11). Note that the texture of $\Omega_\nu$ is essentially the same as that of $M_\nu$ (the first term of $M'_\nu$), and the texture of $\Lambda_\nu$ is essentially

the same as that of the second term of $M'_\nu$. Therefore the basic texture of $\mathbf{M}'_\nu$ is the same as that of $M'_\nu$. In other words, the structure of $M'_\nu$ is stable against radiative corrections.

It is particularly interesting that the second term of $M'_\nu$ and the term $\Lambda_\nu$ in $\mathbf{M}'_\nu$ are possible to cancel eath other. Indeed the cancellation between these two terms takes place, if the coefficient $c'_\nu$ satisfies the condition $c'_\nu/c_\nu = 2\kappa/3$. In this case, the resultant neutrino mass matrix reads as

$$\mathbf{M}''_\nu \approx \left(I_g I_t^6 I_\tau^2\right)\left(M_\nu + 2\kappa\Omega_\nu\right) \ , \tag{14}$$

which is diagonal at the scale $M_Z$. Since the corresponding charged lepton mass matrix $\mathbf{M}'_l$ is of a quasi-democratic form at the same scale, just as $\mathbf{M}_l$ discussed above, the simultaneous diagonalization of $\mathbf{M}'_l$ and $\mathbf{M}''_\nu$ must lead to the democratic flavor mixing pattern $V$ in the lowest order approximation (namely, in the approximation of neglecting the $\Delta V$ term).

One might question whether the condition $c'_\nu/c_\nu = 2\kappa/3 \ll 1$ suffers from fine-tuning or not. Indeed it is rather natural to expect $c'_\nu \ll c_\nu$ in the model under consideration, because this hierarchy assures the near degeneracy of three neutrino masses. On the other hand, a relation between $c'_\nu$ and $c_\nu$ allows us to reduce the number of free parameters in $M'_\nu$ from four to three, which can fully be determined by three neutrino mass eigenvalues $\mathbf{m}'_i$ (for $i = 1, 2, 3$) at low energy scales. Indeed we obtain

$$\begin{aligned}
\mathbf{m}'_1 &\approx \left(I_g I_t^6 I_\tau^2\right)\left(1 + 2\kappa - \delta_\nu\right) c_\nu \ , \\
\mathbf{m}'_2 &\approx \left(I_g I_t^6 I_\tau^2\right)\left(1 + 2\kappa + \delta_\nu\right) c_\nu \ , \\
\mathbf{m}'_3 &\approx \left(I_g I_t^6 I_\tau^2\right)\left(1 + 2\kappa + \varepsilon_\nu\right) c_\nu \ .
\end{aligned} \tag{15}$$

Note that the overall factor $(I_g I_t^6 I_\tau^2)$ takes approximate values 0.80 and 0.63, respectively, for $\tan\beta = 10$ and 60 [4]. The present Super-Kamiokande [2] and CHOOZ [11] data favor the approximate decoupling between solar and atmospheric neutrino oscillations, which are respectively attributed to $\nu_e \rightarrow \nu_\mu$ and $\nu_\mu \rightarrow \nu_\tau$ transitions in the framework of three active neutrinos.Thus one may take $\Delta m^2_{\rm sun} = |(\mathbf{m}'_2)^2 - (\mathbf{m}'_1)^2|$ and $\Delta m^2_{\rm atm} = |(\mathbf{m}'_3)^2 - (\mathbf{m}'_1)^2|$. Taking $\Delta m^2_{\rm sun} \ll \Delta m^2_{\rm atm}$ into account [2], we arrive at

$$\frac{\Delta m^2_{\rm sun}}{\Delta m^2_{\rm atm}} \ \approx \ \frac{2|\delta_\nu|}{|\varepsilon_\nu + \delta_\nu|} \ \approx \ 2\frac{|\delta_\nu|}{|\varepsilon_\nu|} \ . \tag{16}$$

This result implies that the observables $\Delta m^2_{\rm sun}$ and $\Delta m^2_{\rm atm}$ are completely insensitive to the small parameter $\kappa$. Given the energy scale at which the proposed textures of $M'_l$ and $M'_\nu$ hold, $\kappa$ is a well-defined quantity in respect to the fixed value of $\tan\beta$ within the MSSM or other extensions of the standard electroweak model. Therefore we think that the condition $c'_\nu/c_\nu = 2\kappa/3$ is plausible for our new ansatz of lepton mass matrices at a superhigh energy scale, from which the democratic neutrino mixing pattern can be obtained, after radiative corrections, at the experimentally accessible energy scales.

In summary, we have investigated the renormalization effects on lepton mass matrices and flavor mixing from a superhigh energy scale to the electroweak scale in the framework of MSSM. We find that the democratic neutrino mixing pattern may in general be instable against radiative corrections. A new ansatz of lepton mass matrices, based on the breaking of

flavor democracy for charged leptons and the mass degeneracy for light neutrinos, has been prescribed at superhigh scales. Taken the effect of radiative corrections into account, this ansatz can lead to the democratic mixing pattern of lepton flavors at low energy scales. We expect that the forthcoming neutrino oscillation experiments will provide a stringent test of the democratic neutrino mixing pattern and other nearly bi-maximal neutrino mixing schemes, from which one can get more hints to explore possible flavor symmetries and to build viable models of lepton mass matrices at appropriate superhigh energy scales.

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